

High School Mathematics Contest

Elon University Mathematics Department

Saturday, April 2, 2016

1. What is the constant term in the expansion of $\left(2x - \frac{1}{3x}\right)^{14}$?
 - (a) $\left(-\frac{2}{3}\right)^{14}$.
 - (b) $\frac{128}{729}$.
 - (c) $-\frac{128}{729}$.
 - (d) $128 - \frac{1}{729}$.
 - (e) none of the above.
2. What is the number of Real solutions to the equation $\sqrt{x} = 1 + \sqrt{|2x - 7|}$?
 - (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) 4.
3. The medians of a right triangle which are drawn from the vertices of the acute angles are 5 and $\sqrt{40}$. What is the length of the hypotenuse?
 - (a) 7.
 - (b) $\sqrt{52}$.
 - (c) $\sqrt{65}$.
 - (d) $5 + \sqrt{40}/2$.
 - (e) none of the above.
4. Assuming that the following statements are consistent, how many of the statements are true?

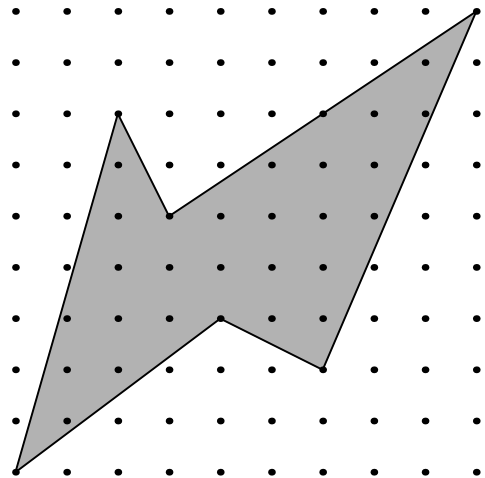
Statement #1: Statements #2 and #3 are true.

Statement #2: Statement #3 is false.

Statement #3: Statement #1 is false.

 - (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) Not enough information to determine.

5. A numerical sample has size 4. The mean is 6, the median is 5.5, and there is a single mode at 4. What is the sample's range?
- (a) 4.
 (b) 5.
 (c) 5.5.
 (d) 6.
 (e) 7.
6. How many divisors does $2^3 3^4 5^6$ have?
- (a) 13.
 (b) 30.
 (c) 48.
 (d) 72.
 (e) 140.
7. How many roots are there to the equation $\tan(100x) = \tan(59x)$ where $0 \leq x \leq 2\pi$?
- (a) 1.
 (b) 41.
 (c) 59.
 (d) 83.
 (e) 100.
8. What the area of the shaded region?



- (a) 22 Sq. Units.
 (b) 24.5 Sq. Units.
 (c) 26 Sq. Units.
 (d) 27 Sq. Units.
 (e) 27.5 Sq. Units.

9. What are the last two digits of $20^{2016} - 16^{2016}$?

- (a) 4.
- (b) 24.
- (c) 44.
- (d) 64.
- (e) 84.

10. Let a and b be two integers whose sum is 565. The following are also true:

- If a is even, $b = 4a + 10$.
- If a is odd, $b = 4a$.

What are a and b ?

- (a) $a = 109, b = 456$.
- (b) $a = 111, b = 454$.
- (c) $a = 112, b = 453$.
- (d) $a = 113, b = 452$.
- (e) $a = 115, b = 450$.

11. A standard 52-card deck is shuffled and a 5-card hand is dealt. What is the probability that the hand contains at least one king?

- (a) 8%.
- (b) 13%.
- (c) 19%.
- (d) 25%.
- (e) 34%.

12. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 2, find the area of the hexagon?

- (a) 3.
- (b) $3\sqrt{2}$.
- (c) $8\sqrt{3}$.
- (d) 6.
- (e) 12.

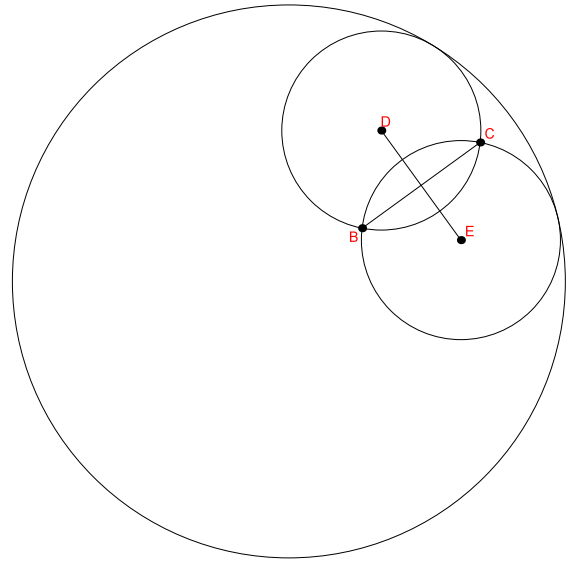
13. Country A has $c\%$ of the world's population and owns $d\%$ of the world's wealth. Country B has $e\%$ of the world's population and owns $f\%$ of the world's wealth. Assume that the citizens of A share wealth equally, and that the citizens of B share wealth equally. Find the ratio of the wealth of a citizen of A to the wealth of a citizen of B.
- (a) $\frac{c}{e}$.
 - (b) $\frac{d}{f}$.
 - (c) $\frac{cd}{ef}$.
 - (d) $\frac{de}{cf}$.
 - (e) none of the above.
14. A ladder is propped up against a wall so that the location the ladder touches the wall is one foot shorter (in distance from the floor) than the total length of the ladder. If this placement of the ladder creates a 60° angle with the floor, what is the distance from the base of the ladder to where the wall meets the floor?
- (a) $2 - \sqrt{3}$ ft.
 - (b) $2 + \sqrt{3}$ ft.
 - (c) $\frac{1}{2 - \sqrt{3}}$ ft.
 - (d) $\frac{1}{2 - \sqrt{3}}$ ft.
 - (e) $\sqrt{3}$ ft.
15. Let a, b, c, d be the complex roots of the polynomial $x^4 + x^2 - 1$. What is the value of $ab + ac + ad + bc + bd + cd$?
- (a) -1 .
 - (b) $-\sqrt{(\sqrt{5} - 1)/2}$.
 - (c) 0 .
 - (d) $\sqrt{-(\sqrt{5} + 1)/2}$.
 - (e) 1 .
16. Let A_1 denote the area of a regular hexagon inscribed in a circle; Let A_2 denote a the area of a regular hexagon circumscribed about that same circle. What is the Ratio of A_1 to A_2 ?
- (a) $\frac{3}{4}$.
 - (b) $\frac{\sqrt{3}}{2}$.
 - (c) $\frac{\pi}{2}$.
 - (d) $\frac{3}{5}$.
 - (e) $\frac{4}{5}$.

17. Solve for x and round to the nearest integer.

$$10^{2x+1} = e^{3x+4}$$

- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) 4.
18. A person travels from point A to Point B with an average speed of 21mph and returns on the same path with a average speed of 42mph. What is the average speed for the entire round trip?
- (a) 28 mph.
 - (b) 29 mph.
 - (c) 30 mph.
 - (d) 31.5 mph.
 - (e) 32 mph.
19. The median to a 10 cm side of a triangle has length 9 cm and is perpendicular to a second median of the triangle. Find the exact value in centimeters of the length of the third median?
- (a) $\sqrt{52}$.
 - (b) $\sqrt{117}$.
 - (c) 13.
 - (d) $\sqrt{182}$.
 - (e) not enough information given.

20. Consider the figure below where two circles are placed within a larger circle. The two interior circles have radii that are half the length of the radius of the larger circle. Line segments BC and DE are of equal length. D and E denote the centers of the smaller two circles. If the radius of the larger circle is denoted r , what is the area of the overlap of the smaller circles?



- (a) $\frac{r^2(\pi - 2)}{8}$.
- (b) $\frac{r^2(\pi - 2)}{4}$.
- (c) $\frac{r^2(\pi - 2)}{2}$.
- (d) $\frac{r^2}{4}$.
- (e) r^2 .
21. Let k be a real number and $f(x) = x^2 + kx \cos(x)$. Suppose further that $f(1) = 5$. What is the value of $f(-1)$?
- (a) -5 .
- (b) -3 .
- (c) -1 .
- (d) 0 .
- (e) 1 .
22. Sally wants to sell toy bears at the fair, and she wants to make as much money as she can. She will charge some price, p , in dollars for each bear. She needs to pay for booth space at the fair and she needs to pay 5 cents times the square of the total number of bears she will bring to sell. What is the most net money in dollars that Sally could leave the fair with?
- (a) $5p^2$.
- (b) $10p$.
- (c) $10p^2$.
- (d) $0.05p^2$.
- (e) p^2 .

23. The parabola $ax^2 + bx + c$ goes through the points $(1, -1)$, $(2,2)$, and $(3,4)$. What is c ?
- (a) -5 .
 - (b) $-1/2$.
 - (c) 0 .
 - (d) 4 .
 - (e) not enough information given.
24. Let x be a real number with $9^x + 9^{-x} = 62$. What is $3^x + 3^{-x}$?
- (a) -8 .
 - (b) $\sqrt{\frac{62 + 16\sqrt{15}}{2}}$.
 - (c) 8 .
 - (d) $\frac{62 + 16\sqrt{15}}{2}$.
 - (e) 64 .
25. Find the area of a rhombus with side of length 13 and one diagonal of length 24.
- (a) 120.
 - (b) $13\sqrt{407}/2$.
 - (c) 156.
 - (d) $156\sqrt{2}$.
 - (e) none of the above.

26. The quadratic $ax^2 + bx + c$ with $a \neq 0$ has integer coefficients and a repeated root. Which of the following can not be a value for ab^2c ?
- (a) 0.
 - (b) 1.
 - (c) 4.
 - (d) 16.
 - (e) 64.
27. In right triangle ACD with right angle at D , B is a point on side AD between A and D . The length of the segment AB is 1. If $\angle DAC = \alpha$ and $\angle DBC = \beta$, find the length of side CD in terms of α and β .
- (a) $\tan(\alpha + \beta)$.
 - (b) $\cot(\alpha + \beta)$.
 - (c) $1/(\tan \alpha - \tan \beta)$.
 - (d) $1/(\cot \alpha - \cot \beta)$.
 - (e) none of the above.
28. What is the surface area of a cube that is inscribed in a Sphere of radius 3?
- (a) 12.
 - (b) 24.
 - (c) $24\sqrt{3}$.
 - (d) 36.
 - (e) 72.
29. Twelve points are equally spaced on the circumference of a circle. How many chords can be drawn that connect pairs of these points and which are longer than the radius of the circle but shorter than its diameter?
- (a) 24.
 - (b) 36.
 - (c) 48.
 - (d) 72.
 - (e) 96.
30. What is the rational equivalent of $\frac{\sqrt{6 + 2\sqrt{5}} - \sqrt{14 + 6\sqrt{5}}}{12}$?
- (a) $-\frac{2}{3}$.
 - (b) $-\frac{1}{3}$.
 - (c) $-\frac{1}{6}$.
 - (d) $\frac{1}{12}$.
 - (e) $\frac{5}{12}$.